

Home Search Collections Journals About Contact us My IOPscience

Depinning, depopulation and crossover effects on tunnelling magnetoplasmons in lateral multiwire superlattices

This article has been downloaded from IOPscience. Please scroll down to see the full text article. 1992 J. Phys.: Condens. Matter 4 1497 (http://iopscience.iop.org/0953-8984/4/6/014)

View the table of contents for this issue, or go to the journal homepage for more

Download details: IP Address: 171.66.16.159 The article was downloaded on 12/05/2010 at 11:16

Please note that terms and conditions apply.

Depinning, depopulation and crossover effects on tunnelling magnetoplasmons in lateral multiwire superlattices

Godfrey Gumbs† and Danhong Huang‡

† Department of Physics, University of Lethbridge, Lethbridge, Alberta TIK 3M4, Canada
 ‡ Department of Physics, Massachusetts Institute of Technology, Cambridge, MA 02139, UK

Received 19 April 1991, in final form 14 October 1991

Abstract. By incorporating lateral tunnelling between adjacent quantum wires, as well as taking into account the dependence of the areal electron density and modulation strength on the varying gate voltage, we have calculated the magnetoplasmon excitation energies of a lateral multiwire superlattice. We have also examined the role played by modulation strength (i.e. barrier height) and applied magnetic field on the collective modes. Our model successfully reproduces several interesting features recently observed with the use of far infrared spectroscopy. Also, a novel commensurability relation between cyclotron and skipping orbits is predicted.

1. Introduction

As a result of the advances made in microlithography, solid-state devices have been fabricated from laterally confined two-dimensional electron gas (2D EG) systems, i.e. quantum wires [1]. Due to the comparable length scales of lateral confinement and the de Broglie wavelength, lateral quantization gives rise to a set of discrete onedimensional (1D) subbands, associated with the free electron motion along the wires [2]. It is well known that, in general, experimental results obtained from far-infrared (FIR) spectroscopy can be correctly explained in terms of the non-local nature of the collective modes [3]. When a perpendicular magnetic field B is applied, Landau quantization and lateral quantization are coupled to one another to form discrete 1D Landau subbands [4]. Consequently, both the subband separation and the electron effective mass are renormalized as a result of the skipping motion of the particle along the sides of the wires.

Recently, Hansen *et al* [1] reported a crossover transition from 2D for a weakly modulated 2D EG to 1D behaviour corresponding to a lateral multiwire superlattice. This is achieved by applying a periodic depletion gate voltage V_g in one direction between the 2D EG and the grating gate. In this experiment both the modulation strength and areal electron density \bar{n}_s are changed by varying V_g . For small negative values of V_g , Hansen *et al* [1] observed a decrease in the excitation energy due to a decrease in \bar{n}_s in the 2D regime [5]. On the other hand, for large negative V_g ($V_g < -0.5 \text{ eV}$), an anomalous increase of the electron energy was observed. Later, we show that this is due to the rapid increase in the lateral quantization in the 1D regime. Furthermore, if V_g is fixed near -0.5 eV on the 1D side we find a depinning of the Fermi energy in the gap, as well as a depopulation of the 1D Landau subbands (LS) with increasing magnetic field B. When B is sufficiently large, we find that the Fermi energy is pinned at the bottom of the 1D LS and the frequency of the collective mode is given by $\omega \sim \omega_c$ corresponding to 2D behaviour. Similar experimental results have been obtained by Heitmann *et al* [6] which were displayed as a sudden change in the slope of $(\hbar\omega)^2$ against B^2 . The theory of Wulf *et al* [5] has been unable to explain the anomalous increase in the excitation energy in the 1D regime. We believe that this failure comes partly from the model they used in which they assumed a cosine potential [7] to simulate strong lateral tunnelling in the 2D regime and may also be due to keeping \bar{n}_s fixed in their calculation while varying V_g . The features observed in the 2D regime (no lateral tunnelling) and make the crossover to the 2D regime by increasing the lateral tunnelling between adjacent quantum wires. In this way, we successfully reproduce the correct results observed in the 1D regime [1].

The rest of this paper is arranged as follows. In section 2, we derive the dispersion relation for a periodic array of quantum wires. In section 3 we present numerical results for the collective magnetoplasmon modes as functions of magnetic field and barrier height.

2. Model for a modulated 2D EG and formalism

For a single quantum wire, in the presence of a perpendicular magnetic field B along the z axis, the Hamiltonian can easily be shown to be

$$\mathcal{H} = \frac{1}{2}m^*\bar{\omega}^2(x+k_y l_B^{*2})^2 - \frac{\hbar^2}{2m^*}\frac{\mathrm{d}^2}{\mathrm{d}x^2} + \frac{\hbar^2 k_y^2}{2m^*(B)} + U_{\mathrm{eff}}(z) \tag{1}$$

where $U_{\rm eff}(z)$ is an effective confining potential for the 2D EG (along the z direction) and a lateral parabolic confining potential is assumed in the x direction. Also, the Landau gauge $A_y = xB$ has been used. $\bar{\omega}^2 = \omega_c^2 + \Omega_0^2, \omega_c = eB/m^*$ is the cyclotron frequency, $l_B^{*2} = (\hbar/eB)(\omega_c/\bar{\omega})$ and the wavevector k_y along the y direction are related to the guiding centre by $x_0 = -k_y l_B^{*2}$. m^* is the effective mass for free electron motion whereas $m^*(B) \equiv m^*(\bar{\omega}/\Omega_0)^2$ is the renormalized effective mass that takes account of the skipping motion on the walls of the wire. Generalizing this to a lateral multiwire superlattice system and assuming a tight-binding model [2], the eigenenergies and eigenfunctions can be expressed as

$$E_n(k_x, k_y) = \left(n + \frac{1}{2}\right)\hbar\tilde{\omega} + \frac{\hbar^2 k_y^2}{2m^*(B)} + \frac{1}{2}\Delta_{n,k_y}(1 - \cos(k_x a))$$
(2)

$$\Psi_{n,k_x,k_y}(x,y,z) = \frac{e^{ik_y y}}{\sqrt{2\pi}} \xi_0(z) \frac{R_n(k_x,k_y)}{\sqrt{N}} \sum_{j=0}^N e^{ik_x j a} \phi_{n,k_y}(x-ja)$$
(3)

where $R_n(k_x, k_y) = [1 + 2\alpha_{n,k_y} \cos(k_x a)]^{-1/2}$ is a normalization factor and $n = 0, 1, 2, \dots, a$ is the separation between adjacent quantum wires. We also have

$$\alpha_{n,k_y} \equiv \int_{-\infty}^{\infty} \mathrm{d}x \, \phi_{n,k_y}^*(x) \phi_{n,k_y}(x-a) \tag{4}$$

and

$$\Delta_{n,k_y} \equiv 4 \int_{-a/2}^{a/2} \mathrm{d}x \,\phi_{n,k_y}^*(x) V_0 \phi_{n,k_y}(x-a) \tag{5}$$

is the bandwidth due to lateral tunnelling, where V_0 is the barrier height which is determined by the gate voltage (see later). In equations (4) and (5) we only include the nearest-neighbour wavefunction overlap. A better treatment should include the wavefunction overlap for all neighbours. However, this presents us with a mathematical problem. Here we limit ourselves to the simplest approximation which is valid only when the wire separation is much larger than the Landau diameter.

$$\xi_0(z) = \frac{1}{\sqrt{2L_z^3}} z e^{-z/2L_z}$$
(6)

is a variational-type of wavefunction, representing the quantum confinement in the z direction and L_z is the thickness of the 2D layer. $\phi_{n,k_y}(x)$ is the wavefunction for a harmonic oscillator and is given by

$$\phi_{n,k_y}(x) = \left[\frac{1}{\pi^{1/2} 2^n n! l_0^*}\right]^{1/2} \exp\left(\frac{(x+k_y l_B^{*2})^2}{2l_0^{*2}}\right) H_n\left(\frac{x+k_y l_B^{*2}}{l_0^*}\right).$$
(7)

In our notation, l_0^* is a renormalized Landau radius defined by $l_0^{*2} = l_0^2(\Omega_0/\bar{\omega})$ where l_0 is the lateral extension of the lowest eigenstate at B = 0.

With the use of standard many-body theory [8, 9], the dispersion relation for magnetoplasmons can be obtained in the random-phase approximation (RPA). The collective modes correspond to the solutions of

$$Det[\delta_{jj'} - U(q_x + j'G, q_y)P(q_x + jG, q_x + j'G, q_y, \omega; (j - j'))] = 0$$
(8)

where $G = 2\pi/a$ is a reciprocal lattice vector along the x direction. The Fourier transform of the Coulomb potential is

$$U(q_x + j'G, q_x) = \frac{4\pi e^2}{\epsilon_s} \frac{I([(q_x + j'G)^2 + q_y^2]^{1/2}L_z)}{[(q_x + j'G)^2 + q_y^2]^{1/2}}$$
(9)

with the screening factor, due to the finite thickness of the 2D layer, determined by [8]

$$I(qL_z) = \frac{8 + 9qL_z + 3(qL_z)^2}{8(1 + qL_z)^3}.$$
(10)

The density-density response function is defined by

$$P(q_{x} + jG, q_{x} + j'G, q_{y}, \omega; (j - j')) \equiv \sum_{n' \ge n} \sum_{k_{x}, k_{y}} |R_{n'}(k_{x} + q_{x})|^{2} |R_{n}(k_{x})|^{2}$$

$$\times \prod_{nn'}(q_{x}, q_{y}, k_{x}, k_{y}, \omega; (j - j'))$$

$$\times A_{nn'}(q_{x} + jG, q_{y}, k_{x}, k_{y}) A_{nn'}^{*}(q_{x} + j'G, q_{y}, k_{x}, k_{y})$$
(11)

where the form factor $A_{nn'}$ is defined by

$$\begin{aligned} A_{nn'}(q_{x} + jG, q_{y}, k_{x}, k_{y}) &\equiv \sqrt{\frac{1}{2^{n'-n}}} \sqrt{\frac{n!}{n'!}} \\ &\times \exp\left[i\left(k_{y} + \frac{q_{y}}{2}\right)q_{x}l_{B}^{*2}\right] \exp\left\{-\frac{1}{4}\left[q_{y}^{2}l_{B}^{*2}\left(\frac{l_{B}^{*}}{l_{0}^{*}}\right)^{2} + q_{x}^{2}l_{0}^{*2}\right]\right\} \\ &\times \sum_{m=-1}^{+1} \exp\left\{-\frac{1}{4}\left[\left(\frac{ma}{l_{0}^{*}}\right)^{2} + 2\left(\frac{ma}{l_{0}^{*}}\right)\left(\frac{l_{B}^{*}}{l_{0}^{*}}\right)q_{y}l_{B}^{*}\right]\right\} \\ &\times \left[q_{y}l_{B}^{*}\left(\frac{l_{B}^{*}}{l_{0}^{*}}\right) - iq_{x}l_{0}^{*} + \left(\frac{ma}{l_{0}^{*}}\right)\right]^{n'-n} \\ &\times L_{n}^{n'-n}\left[\frac{1}{2}\left\{\left[q_{y}l_{B}^{*}\left(\frac{l_{B}^{*}}{l_{0}^{*}}\right) + \left(\frac{ma}{l_{0}^{*}}\right)\right]^{2} + q_{x}^{2}l_{0}^{*2}\right\}\right] \\ &\times \exp\left[-i\left(k_{x} + \frac{q_{x}}{2}\right)ma\right]. \end{aligned}$$
(12)

The special function $L_n^{n'-n}(x)$ is an associated Laguerre polynomial and the irreducible polarizability depends on the umklapp process and is given by

$$\Pi_{nn'}(q_x, q_y, k_x, k_y, \omega; (j-j')) = \exp[ik_y Gl_B^*(j-j')] \{\theta[k_y + k_{yF}^{(n')}(k_x + q_y) + q_y] \\ \times \theta[k_{yF}^{(n')}(k_x + q_x) - q_y - k_y] - \theta[k_{yF}^{(n)}(k_x) - |k_y|] \} \\ \times \{(\hbar^2 q_y k_y / m^*(B)) + [(\hbar^2 q_y^2 / 2m^*(B)) + \Delta E_{nn'}(k_x, q_x) - \hbar\omega]\}^{-1} \\ + (1 - \delta_{nn'})(\omega \to -\omega).$$
(13)

In this notation, a similar term with opposite sign in frequency must be added to the first and $\theta(x)$ is the unit step function. Also

$$\Delta E_{nn'}(k_x, q_x) = (n' - n)\hbar\bar{\omega} + \frac{1}{2}(\Delta_{n'} - \Delta_n) + \frac{1}{2}[\Delta_n \cos(k_x a) - \Delta_{n'} \cos(k_x + q_x)a].$$
(14)

Making use of the property that the wavefunction $\phi_{n,k_y}(x)$ is localized, it is reasonable to approximate the bandwidth defined in equation (5) by

$$\Delta_{n,k_y} \approx 4 \int_{-\infty}^{\infty} dx \, \phi_{n,k_y}^*(x) V_0 \phi_{n,k_y}(x-a) = 4 \alpha_n V_0$$
$$= 4 V_0 L_n^0 (a^2/2l_0^{*2}) e^{-a^2/4l_0^{*2}}$$
(15)

whenever $k_y^2 l_B^{*2} \ll 1$. Equation (15) implies that the tight-binding model corresponds to $a > 2l_0^*$. There is also a commensurability relation between the cyclotron and skipping motions for the strongly modulated system [7] that corresponds to the zeros of the Laguerre polynomial $L_n^0(x)$ for $n \gg 1$:

$$2\mathcal{R}_n\left(\frac{G}{n}\right) = \frac{8}{(\lambda - \frac{1}{4})} \qquad \lambda = 1, 2, \dots$$
(16)

Clearly this relation depends on the subband index n but not on the guiding centre x_0 . Here, $\mathcal{R}_n = \sqrt{2n+1}l_0^*$ is the renormalized cyclotron radius. $k_{yF}^{(n)}(k_x)$ is the Fermi wavenumber for the nth level. We have

$$k_{yF}^{(n)}(k_x) = \sqrt{\frac{2m^*(B)}{\hbar^2}} \left[E_F - \left(n + \frac{1}{2}\right)\hbar\bar{\omega} - \frac{\Delta_n}{2}(1 - \cos(k_x a)) \right]^{1/2}$$
(17)

if

$$E_{\mathrm{F}} \geqslant \left(n + \frac{1}{2}\right) \hbar \bar{\omega} + \frac{\Delta_n}{2} (1 - \cos(k_x a))$$

and zero otherwise. For a chosen value of \bar{n}_s , the Fermi energy E_F must be determined self-consistently. We incorporate the effect of a varying gate voltage on \bar{n}_s and V_0 through the following model. For this, we observe that the 1D channel width W has been calculated as [4]

$$W = 2\pi n_{1D}^{1/3} \left[\frac{2\hbar}{3\pi m^* \Omega_0} \right]^{2/3}$$
(18)

where the n_{1D} is the linear electron density and the areal density is given by [1]

$$\bar{n}_{s} = \begin{cases} [\Omega_{0}^{(th)} / \Omega_{0}]^{2/3} n_{c} & \text{for } \Omega_{0} \ge \Omega_{0}^{(th)} \\ [\Omega_{0}^{(0)} / \Omega_{0} + \Omega_{0}^{(0)}]^{2/3} n_{0} & \text{for } 0 \le \Omega_{0} \le \Omega_{0}^{(th)} \end{cases}$$
(19)

where

$$\Omega_0^{(th)} = (2\pi)^{3/2} n_c^{1/2} \left(\frac{2\hbar}{3\pi m^* a}\right)$$
(20a)

$$\Omega_0^{(0)} = \Omega_0^{(th)} \left[\frac{n_c}{n_0 - n_c} \right]^{3/2}.$$
(20b)

 n_c is the threshold areal electron density that separates 1D and 2D behaviour and n_0 is the areal electron density when the gate voltage is zero. We define the barrier height (or modulation strength) by

$$V_0 = \frac{1}{2} m^* \Omega_0^2 \left(\frac{W}{2}\right)^2 = \frac{1}{3} \left[\frac{2\pi^2 \hbar^2 \bar{n}_s^2 a^2 (\hbar\Omega_0)^2}{3m^*}\right]^{1/3}$$
(21)

which is explicitly dependent on \bar{n}_s . Making use of these results, we have self-consistently calculated the Fermi energy by solving

$$\bar{n}_{s} = \frac{a\sqrt{2m^{*}(B)}}{\pi^{2}\hbar} \sum_{n}^{\text{occupied}} \int_{-\pi/a}^{\pi/a} \mathrm{d}k_{x}$$

$$\times \left[E_{\mathrm{F}} - \left(n + \frac{1}{2}\right)\hbar\bar{\omega} - \frac{\Delta_{n}}{2}(1 - \cos(k_{x}a)) \right]^{1/2}. \tag{22}$$

The soundness of our model is ensured by demanding that $|x_0| \leq a/2$ which means that for every value of k_x we must have

$$\Omega_0 \ge \Omega_0^{(\min)} \equiv \frac{\hbar}{2m^*a} \left(\frac{\omega_c}{\bar{\omega}}\right) \left[E_F - \frac{1}{2}\hbar\bar{\omega} - \frac{\Delta_0}{2}(1 - \cos(k_x a)) \right]^{1/2}.$$
(23)

This restriction may give rise to a cut-off of the k_x integral in equation (22). Keeping Ω_0 , we find that in the quantum strong field limit ($\omega_c \gg \Omega_0$) there is complete Landau quantization for which the kinetic energy is quenched and the energy eigenvalues become k_y -independent with

$$E_n(k_x) = \left(n + \frac{1}{2}\right)\hbar\omega_c + \frac{\Delta_n}{2}(1 - \cos(k_x a)).$$
⁽²⁴⁾

Both the depopulation of the energy levels and the pinning of the Fermi energy will evolve in this regime. On the other hand, when $\hbar\omega_c \ll \hbar\Omega_0$, the scaled Fermi energy $E_{\rm F}/\hbar\bar{\omega}$ is independent of magnetic field. When $\hbar\omega_c > \hbar\Omega_0$ there is a depinning of the Fermi energy. For a fixed magnetic field, $\bar{n}_{\rm s}$ will be reduced when $V_{\rm g}$ changes from zero to negative values and the barrier height increases correspondingly. For $\hbar\Omega_0 \ll \hbar\omega_c$, we find that $E_{\rm F}/\hbar\bar{\omega}$ is independent of $\hbar\Omega_0$ in this 2D regime. However, due to an increase in lateral tunnelling when V_0 is reduced, the tunnelling 1D LS still depopulate. Furthermore, when the separation *a* between adjacent quantum wires is increased ($a \gg 2l_0^*$, which is equivalent to an increase in magnetic field or modulation strength) interwire interaction and lateral tunnelling are suppressed and the system behaves like a single quantum wire.



Figure 1. Calculated magnetoplasmon excitation energy as a function of magnetic field. In these calculations we took $m^* = 0.07 m_e$, $\epsilon_r = 13.6$ ($\epsilon_s = 4\pi\epsilon_0\epsilon_r$), $n_c = 1.8 \times 10^{11} \text{ cm}^{-2}$, $n_0 = 6.0 \times 10^{11} \text{ cm}^{-2}$, $L_z = 50$ Å, $q_x = q_y = 0$, $\hbar\Omega_0 = 4.0$ meV, a = 150 nm.



Barrier Height V_0 (meV)

Figure 2. Calculated magnetoplasmon excitation energy as a function of V_0 . The values for $m^*, \epsilon_r, n_c, n_0, L_z$ and a are the same as those used for figure 1. Here B = 2.19 T and $q_x = q_y = 0$. The inset shows V_0 and E_F as functions of the confinement energy $\hbar\Omega_0$. The threshold value of $\hbar\Omega_0^{th}$ is indicated by a vertical arrowhead. The 1D and 2D regimes correspond to the right- and left-hand sides of the arrow, respectively.

3. Numerical results

There are three types of collective excitations for the quantum wire array. There are: (1) intersubband excitations (n = n' where the nth ID LS is occupied); (2) intersubband excitations $(n \neq n' \text{ where the } n\text{th ID LS is occupied})$ but the n'th ID LS is occupied; and (3) edge-state excitations $(n \neq n' \text{ where both } n \text{ and } n' \text{ ID LS are occupied})$. Due to lateral tunnelling, the edge-states with (large momentum transfer) $q_y = \pm (k_{yF}^{(0)} + k_{yF}^{(1)})$ and the intrasubband excitations with $q_y = \pm 2k_{yF}^{(0)}$ can give large contributions to the excitation energy, compared with those modes excited in the absence of any lateral tunnelling.

In figure 1 we have plotted the square of the excitation energy as a function of the square of magnetic field. Within the range $B^2 = 1.0 \text{ T}^2 \sim 2.0 \text{ T}^2$, we find a sudden change of slope associated with the depinning of the Fermi energy in the energy gap. In this range of magnetic fields, E_F has a peak. On further increasing B, depopulation of the ID LS will follow which is accompanied by some peaks in the E_F curve. Similar 'kinks' are expected to be repeated. This feature will be enhanced when fewer ID LSS are occupied, i.e. in the high magnetic field regime. When $\hbar\omega_c \gg \hbar\Omega_0$, we expect a linear dependence. It might be possible to confirm this experimentally [6].

Figure 2 shows a plot of the plasmon excitation energy $\hbar\omega$ as a function of V_0 . For small values of V_0 we find an increase in energy as V_0 decreases due to an increase in \bar{n}_{s} . When $V_{0} \ge 63.6$ meV, we enter into the 1D regime. In this region, we have found an anomalous decrease in the energy of the collective mode with V_0 . From the inset in figure 2 we could deduce the threshold value $\bar{n}_s = n_c$ which corresponds to the local minimum in the excitation energy. We also observe the effects due to depinning of the Fermi energy and depopulation of the 1D LS, as is evident in figure 1. Each depinning or depopulation is accompanied by a peak in the Fermi energy curve. It might be possible to confirm this from the experiments carried out in [1]. Direct comparison between our results for the barrier height dependence of the excitation energy and the experimental data in [1] is not a straightforward matter. Of course, the barrier height is not the same as the gate voltage. In order to present a quantitative comparison between theory and experiment, we need to know the functional relation between the gate voltage and the areal electron density. However, this is a non-trivial problem and may only be determined experimentally. Moreover, we expect that our equation (19) will only be suitable in the 1D regime as well as in a very restricted region of the 2D regime lying close to the threshold. We believe that the method used in [5] to scale the gate voltage with the potential barrier height depends on the assumption that there is a linear relation between them. It is not clear that this would be valid for the entire range of values of gate voltage. From a physical point of view, the barrier height should increase with the gate voltage. Therefore, the general trend displayed in figure 2 would persist when plotted as a function of gate voltage. We emphasize that the aim of our paper is to explain qualitatively some of the interesting features found in the FIR experiments.

In conclusion, our theory has successfully explained some interesting features recently observed in FIR experiments. Combining our results with those obtained with a cosine-modulation potential in the 2D regime, we are now able to get a complete understanding of magnetoplasmons for the ID, 2D and crossover regions.

Acknowledgments

This work was supported in part by the Natural Sciences and Engineering Research Council of Canada.

References

- [1] Hansen W, Horst M, Kotthaus J P, Merkt U, Sikorski Ch and Ploog K 1987 Phys. Rev. Lett. 58 2586
- [2] Li Q and Das Sarma S 1990 Phys. Rev. B 41 12068
- [3] Egeler T, Abstreiter G, Weimann G, Demel T, Heitmann D and Grambow P 1990 Phys. Rev. Lett. 65 1804
- [4] Hirler F, Smoliner J, Gornik E, Weimann G and Schlapp W 1990 Appl. Phys. Lett. 57 261 Gerhardts R R, Weiss D and Klizing K V 1989 Phys. Rev. Lett. 62 1173 Winkler R W, Kotthaus J P and Ploog K 1989 Phys. Rev. Lett. 62 1177
- [5] Wulf U, Zeeb E, Gies P, Gerhardts R R and Hanke W 1990 Phys. Rev. B 42 7637
 [6] Heitmann D unpublished
- [7] Cui H L, Fessatidis V and Horing M J M 1989 Phys. Rev. Lett. 63 2598 Zhang C 1990 Phys. Rev. Lett. 65 2207
- [8] Huang D and Zhou S X 1990 J. Phys.: Condens. Matter 2 501
- [9] Gumbs G and Zhu X 1989 Solid State Commun. 70 389